

A Utility-Based Distributed Maximum Lifetime Routing Algorithm for Wireless Networks

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Abstract—Energy-efficient routing is a critical problem in multihop wireless networks due to the severe power constraint of wireless nodes. Despite its importance and many research efforts toward it, a distributed routing algorithm that maximizes network lifetime is still missing. To address this problem, this paper proposes a novel utility-based nonlinear optimization formulation to the maximum lifetime routing problem. Based on this formulation, a fully distributed localized routing algorithm is further presented, which is proved to converge at the optimal point, where the network lifetime is maximized. Solid theoretical analysis and simulation results are presented to validate the proposed solution.

Index Terms—Energy, non-linear optimization, utility, wireless network.

I. INTRODUCTION

MULTIHOP wireless networks can be formed by wireless nodes with no preexisting and fixed infrastructures. In order to provide communication throughout the network, wireless nodes cooperate to handle network functions, such as packet routing. One example of a multihop wireless network is a sensor network, which can be readily deployed in diverse environments, such as health care, military, and disaster detection, to collect and process useful information in an autonomous manner.

One important issue in wireless networks is the energy constraint—wireless nodes carry limited and irreplaceable power supply. Moreover, radio communication consumes a large fraction of this supply. Such observations pose critical demand to design energy-efficient packet routing algorithms. In order to scale to larger networks, such algorithms need to be localized. The key design challenge is to derive the desired global system properties in terms of energy efficiency from the localized algorithms.

In existing works, the problem of designing energy-efficient routing algorithms has been extensively studied in both general multihop wireless networks [1]–[5], and the particular backdrop of sensor networks [6]–[10]. Various goals may be achieved by

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these energy-efficient routing algorithms, such as minimizing energy consumption for end-to-end paths [4], [11]–[16] or maximizing the lifetime of the whole network [1]–[3], [6], [17]. Here, we give a brief overview of these existing approaches from an optimization theoretical viewpoint and highlight the original contribution of this work in light of previous works.

A. Minimum Energy Routing: User Optimization

The minimum energy routing problem presents a “user optimization” problem. It tries to optimize the performance of a single user (an end-to-end connection), minimizing its energy consumption. To solve this problem, the typical approach [14], [15] is to use a shortest path algorithm in which the edge cost is the power consumed to transmit a packet between two nodes of this edge. Although effectively reducing the energy consumption rate, this approach can cause unbalanced consumption distribution, i.e., the nodes on the minimum-energy path are quickly drained of energy, causing network partition or malfunctioning. Some routing algorithms [4], [12] associate a cost with the node of low energy reserve, but remain to be heuristic solutions.

B. Maximum Lifetime Routing: System Optimization

The maximum network lifetime routing problem tries to maximally prolong the duration in which the entire network properly functions. It presents a “system optimization” problem, which is radically different from “user optimization.” To achieve the goal of “system optimization,” global coordination is required, which poses a significant challenge to the design of a distributed routing algorithm. On the other hand, maximum lifetime routing well addresses the power consumption balance problem of the minimum energy routing, a more critical issue of wireless networks.

1) *Existing Approaches—Linear Optimization Formulation:* The inherent characteristic of the maximum lifetime routing problem naturally leads to a linear optimization formulation [1]–[3], [6], [17]. A combinatorial algorithm can then be designed to solve this problem. In particular, the following algorithms have been presented in the existing work.

Heuristic: Heuristic algorithms are presented in [2] and [6]. Being heuristics, these algorithms lack solid theoretical proof of their performance. For example, the algorithm in [2] may have degraded worst case performance, and the algorithm in [6] does not scale well to the network size in terms of running time.

Centralized approximation algorithm: In [3], Chang and Tassiulas adopt a classical linear optimization approach—the Garg–Konemann [18] algorithm—for multicommodity flow and provide a centralized combinatorial approximation solution. Being a centralized algorithm, it is hard to be deployed on realistic wireless network environments.

Distributed combinatorial algorithm: In a recent work, Sankar and Liu [1] adopt a distributed flow algorithm due to Awerbuch and Leighton [19], [20]. Yet, the core of this algorithm can only verify whether a traffic input can be satisfied by a required network lifetime, and if so, how to route traffic. To calculate the exact network lifetime, a bisection search is needed. Thus, to deploy this algorithm to route packets online, it will suffer from slow convergence and potential performance fluctuation due to the bisection search.

2) *Our Approach—Nonlinear Optimization:* In this paper, we aim at designing a fully distributed routing algorithm that achieves the goal of maximizing network lifetime. Toward this goal, we propose a utility-based nonlinear optimization formulation of the maximum lifetime routing problem. The essential idea of this formulation is based on the following observation. Our goal is to maximize the lifetime of the node that has the minimum lifetime among all nodes. If we regard lifetime as a “resource,” then this goal can be regarded as to “allocate lifetime” to each node so that the max–min fairness criterion is satisfied. This “lifetime allocation” mechanism needs to be achieved via routing and has to satisfy the traffic demand constraint. From this view, we further adopt the concept of “utility” that has been widely used in resource allocation in economics as well as distributed computing. By defining an appropriate utility function based on lifetime, the problem of achieving the max–min lifetime allocation is converted into the aggregated utility maximization problem.

Based on this formulation, the key to the distributed algorithm is to consider the marginal utility at each node. We show that the system achieves optimal routing when the marginal utilities of all nodes are equal. Hence, the design philosophy of a distributed routing algorithm is to let each node balance its own marginal utility by iteratively adjusting traffic on different routing paths. Such a design philosophy was first proposed by Gallager [22] in his seminal work to minimize the overall network delay through distributed routing. Gallager’s algorithm was later improved by Bertsekas *et al.* [21] to speed up the convergence speed. Although sharing the same design thought, our problem has entirely different optimization goals with different objective functions.

To summarize, despite the importance of the maximum network lifetime problem and many research efforts toward it, a fully distributed localized on-line routing algorithm is still missing. Toward this goal, this paper makes the following original contributions. First, we present a unique utility-based nonlinear optimization formulation of the maximum lifetime routing problem. Second, based on this formulation, we propose a fully distributed and localized routing algorithm with solid theoretical analysis of its optimality. To the best of our knowledge, this is the first localized maximum lifetime routing algorithm. It is important to note that, although designed for energy-efficient routing in wireless networks, the presented al-

gorithm can also be extended to other scenarios where a global optimization goal needs to be achieved by distributed routing.

The rest of this paper is organized as follows. We model the network in Section II and present the utility-based formulation of the network lifetime maximization problem in Section III. Sections IV and V present the optimality condition of the solution and the distributed maximum network lifetime routing algorithm and its analysis. Section VI shows the performance study, Section VII presents related work, and Section VIII concludes this paper. Due to space limitations, all the proofs can be found in our technical report [23].

II. MODEL

We consider a multihop wireless network that consists of a set of wireless nodes, represented as $\mathcal{N} = \{1, 2, \dots, n\}$. Two nodes that are within the transmission range of each other can communicate directly and form a wireless link. Let \mathcal{L} be the set of wireless links, denoted as $\mathcal{L} = \{(i, k) \mid \text{a wireless link goes from } i \text{ to } k\}$. Each link (i, k) has a weight d_{ik} , which is the distance between the antennas of nodes i and k .

To illustrate how traffic is routed in the network, we further introduce the following definitions and notations.

- input traffic $r_i(j) \geq 0$: the traffic (in bits per second) generating at node i and destined for node j ;
- node flow $t_i(j)$: the total traffic at node i destined for node j . $t_i(j)$ includes both $r_i(j)$ and the traffic from other nodes that is routed through i to destination j ;
- routing variable $\phi_{ik}(j)$: the fraction of the node flow $t_i(j)$ routed over link (i, k) . This variable defines a routing solution. It is obvious that
 - $\phi_{ik}(j) = 0$, if $(i, k) \notin \mathcal{L}$, as no traffic can be routed through a nonexistent link;
 - $\phi_{ik}(j) = 0$, if $i = j$, because traffic that has reached its destination is not sent back into the network;
 - as node i must route its entire node flow $t_i(j)$ through all outgoing links

$$\sum_{k \in \mathcal{N}} \phi_{ik}(j) = 1, \quad \forall i, j \in \mathcal{N}.$$

We illustrate the above concept in Fig. 1. In the figure, the input traffic is $r_2(4) = 2$ kb/s, $r_3(6) = 3$ kb/s, and $r_1(6) = 1$ kb/s. The node flows are $t_2(4) = r_2(4)$, $t_3(6) = r_3(6)$, $t_1(4) = 2$ kb/s, $t_1(6) = 4$ kb/s, $t_4(6) = 1$ kb/s, and $t_5(6) = 3$ kb/s. The routing variables are as follows: $\phi_{21}(4) = \phi_{31}(6) = \phi_{14}(4) = \phi_{46}(6) = \phi_{56}(6) = 1$, $\phi_{14}(6) = 1/4$, and $\phi_{15}(6) = 3/4$.

One important constraint of traffic routing in a network is flow conservation: the traffic into a node for a given destination is equal to the traffic out of it for the same destination. This constraint can be formally expressed as

$$t_i(j) = r_i(j) + \sum_{l \in \mathcal{N}} t_l(j) \phi_{li}(j), \quad \forall i, j \in \mathcal{N}. \quad (1)$$

We can also understand the routing problem as a multicommodity problem, i.e., the flow to each destination node j is regarded as a commodity j . Then, the above statement simply

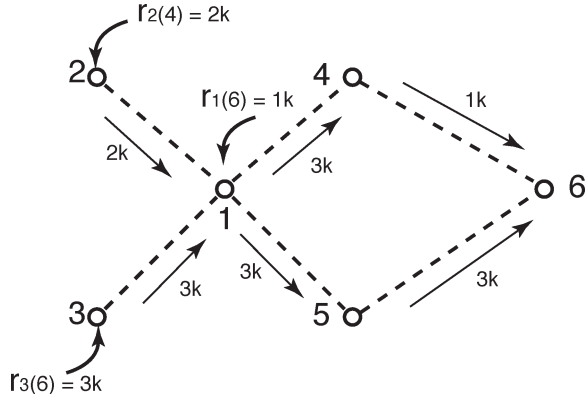


Fig. 1. Example network and traffic.

means that the amount of a commodity that enters and leaves a node must be the same.

Now we present the energy model used in this paper. We define E_i as the energy reserve at node i . Let p_i^r (joules per /bit) be the power consumption at node i , when it receives one unit of data, and p_{ik}^t (J/bit) be the power consumption when one unit of data is sent from i over link (i, k) . Based on the first-order radio model, we have

$$p_i^r = \alpha \quad (2)$$

$$p_{ik}^t = \alpha + \beta d_{ik}^m \quad (3)$$

where α is a distance-independent constant that represents the energy consumption to run the transmitter or receiver circuitry, and β is the coefficient of the distance-dependent term that represents the transmit amplifier. The exponent m is determined from field measurements, which is typically a constant between 2 and 4.

The routing variables $\phi_{ik}(j) \forall j$ and the node flow set $t_i(j) \forall j$ jointly determine the traffic sent along a wireless link (i, k) . Further, $t_i(j) \forall j$ and the input traffic $r_i(j) \forall j$ jointly determine the traffic received at node i . Thus, we have the wireless node i 's power consumption rate p_i in joules per second as

$$p_i = \sum_{j \in \mathcal{N}} \left[t_i(j) \sum_{k \in \mathcal{N}} p_{ik}^t \phi_{ik}(j) + p_i^r (t_i(j) - r_i(j)) \right]. \quad (4)$$

We summarize above notations into the following sets:

- input set: $\mathbf{r} = \{r_i(j) | i, j \in \mathcal{N}\}$;
- node flow set: $\mathbf{t} = \{t_i(j) | i, j \in \mathcal{N}\}$;
- routing variable set: $\phi = \{\phi_{ik}(j) | i, j, k \in \mathcal{N}\}$;
- power consumption set: $\mathbf{p} = \{p_i | i \in \mathcal{N}\}$.

Note that the relations among the input set and the node flow set are constrained by flow conservation. We further have the following lemma.

Lemma 1: Given the input set \mathbf{r} and routing variable set ϕ , the set in (1) has a unique solution for \mathbf{t} . Each element $t_i(j)$ is nonnegative and continuously differentiable as a function of \mathbf{r} and ϕ .

The detailed proofs of lemmas and theorems in this paper can be found in the technical report.

III. PROBLEM FORMULATION

Maximum network lifetime routing tries to maximally prolong the duration in which the entire network properly functions. Here we consider the network lifetime as the lifetime of the wireless node who dies first.¹ The problem of maximum network lifetime routing asks the given traffic demand how to route the traffic so that the network lifetime can be maximized. Let T_i denote the lifetime of wireless node i , and T as the lifetime of the wireless network, i.e., $T = \min\{T_i, i \in \mathcal{N}\}$. The problem can be formulated as a linear optimization problem

$$\begin{aligned} & \text{maximize} && T \\ & \text{subject to} && p_i T \leq E_i \quad \forall i \in \mathcal{N}. \\ & && \text{Eq. (1) and Eq. (4)} \end{aligned} \quad (5)$$

In this formulation, (5) comes from the definition of lifetime, i.e., the energy consumption at any node i within the network lifetime is no more than its energy reserve, i.e.,

$$p_i T \leq p_i T_i = E_i, \quad \forall i \in \mathcal{N}.$$

Equations (1) and (4) represent the flow conservation and the power consumption rate, respectively. For a given network layout $(\mathcal{N}, \mathcal{L})$ and traffic input set \mathbf{r} , one can acquire the optimal routing solution ϕ to maximize T by solving the above linear optimization problem via a centralized algorithm (e.g., [18]). The real challenge is how to solve this problem in a distributed fashion.

To address this challenge, we propose a novel utility-based nonlinear optimization formulation of the maximum lifetime routing problem, which can lead to a fully distributed routing algorithm. This formulation is inspired by the max–min resource allocation problem in distributed computing and networking areas. Max–min fairness means that for any user i , increasing its resource share x_i cannot be achieved without decreasing the resource share of another user x_j that satisfies $x_i \geq x_j$. Simply put, the max–min allocation mechanism maximizes the resource share of the user who is allocated with the minimum resource.

In the context of maximum life time routing, if we regard lifetime of a node as a certain ‘‘resource’’ of its own, then the goal of maximizing network lifetime can be regarded as to ‘‘allocate lifetime’’ to each node so that the max–min fairness criterion is satisfied. This ‘‘lifetime allocation’’ mechanism needs to be achieved via routing and has to satisfy the traffic input, flow conservation, and power consumption constraints.

We further adopt the concept of ‘‘utility’’ that has been widely used in the area of resource allocation. Defined on the resource share of a user, utility usually represents the degree of satisfaction of this user. It is shown in [25] and [26] that by defining an appropriate utility function, the problem of achieving max–min fairness can be converted into the problem of maximizing the

¹We also note that there are other definitions of network lifetime, such as α -lifetime [24], to which the presented formulation and algorithm can be extended.

aggregated utility (sum of utilities of all users). Thus, we define the utility U_i of a node i as a function of its lifetime T_i as

$$U_i(T_i) = \frac{T_i^{1-\gamma}}{1-\gamma}, \quad \gamma \rightarrow \infty \quad (6)$$

where $T_i = E_i/p_i$ and γ can be made arbitrarily large to infinite. How to determine the value of γ and its impact will be discussed in detail in Section VI. It is shown that [25], [26] T_i , $i \in \mathcal{N}$ satisfy the max-min fair if and only if it solves the aggregated utility maximization problem $\max \sum_{i \in \mathcal{N}} U_i$, with U_i defined as in (6).

To this end, we reformulate the problem of maximum network lifetime as to maximize the aggregate utility of all nodes within the network, i.e.,

$$\begin{aligned} \mathbf{U} : \text{maximize} \quad & U = \sum_{i \in \mathcal{N}} U_i \\ \text{subject to} \quad & p_i T_i \leq E_i \quad \forall i \in \mathcal{N} \\ & \text{Eq. (1) and Eq. (4)} \end{aligned}$$

IV. OPTIMALITY CONDITIONS

With the presented utility-based problem formulation, we only need to seek a distributed algorithm that solves the problem \mathbf{U} in order to achieve maximum lifetime routing. First, we need to understand the optimality condition of such a solution. From the nonlinear optimization theory [27], we consider the first-order conditions in problem \mathbf{U} . Note that the utility U_i is a function of node lifetime T_i , which directly associates with p_i based on relation $T_i = E_i/p_i$. Thus, we can write $U_i(p_i)$ as a function of p_i . Power consumption p_i in turn depends on the input set \mathbf{r} and the routing variable set ϕ . Thus, we calculate the partial derivatives of the aggregate utility U with respect to the inputs \mathbf{r} and the routing variables ϕ , respectively.

We first consider $\partial U / \partial r_i(j)$, the marginal utility on node i with respect to commodity j . Assume that there is a small increment ϵ on the input traffic $r_i(j)$. Then the portion $\epsilon \phi_{ik}(j)$ from this new incoming traffic will flow over the wireless link (i, k) . This will cause an increment power consumption $\epsilon \phi_{ik}(j) p_{ik}^t$ on node i in order to send out the incremented traffic. And the consequent utility change of node i is²

$$\epsilon \phi_{ik}(j) p_{ik}^t U_i'(p_i).$$

On the receiver side, this will cause an increment power consumption $\epsilon \phi_{ik}(j) p_k^r$ on node k in order to receive the incremented traffic. The consequent utility change of node k is

$$\epsilon \phi_{ik}(j) p_k^r U_k'(p_k).$$

If node k is not the destination node, then the increment $\epsilon \phi_{ik}(j)$ of extra traffic at node k will cause the same utility change onward as a result of the increment $\epsilon \phi_{ik}(j)$ of input traffic at node k . To first order this utility change will be

$\epsilon \phi_{ik}(j) \partial U / \partial r_k(j)$. Summing over all adjacent nodes k , then, we find that

$$\begin{aligned} \frac{\partial U}{\partial r_i(j)} &= \sum_{k \in \mathcal{N}} \phi_{ik}(j) \left[p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k) + \frac{\partial U}{\partial r_k(j)} \right] \\ &= \sum_{k \in \mathcal{N}} \phi_{ik}(j) \left[U_{ik}' + \frac{\partial U}{\partial r_k(j)} \right] \\ &= \sum_{k \in \mathcal{N}} \phi_{ik}(j) \delta_{ik}(j) \end{aligned} \quad (7)$$

where $U_{ik}' = p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k)$ is called the marginal utility on link (i, k) , and $\delta_{ik}(j) = U_{ik}' + \partial U / \partial r_k(j)$ is called the marginal utility of link (i, k) with respect to commodity j .

Equation (7) asserts that the marginal utility of a node is the convex sum of the marginal utilities of its outgoing links with respect to the same commodity. By the definition of ϕ , we can see that $\partial U / \partial r_j(j) = 0$, since $\phi_{jk}(j) = 0$, i.e., no traffic of commodity j needs to be routed anymore once it arrives at the destination.

Next we consider $\partial U / \partial \phi_{ik}(j)$. An increment ϵ in $\phi_{ik}(j)$ causes an increment $\epsilon t_i(j)$ in the portion of $t_i(j)$ flowing on link (i, k) . If $k \neq j$, this causes an addition $\epsilon t_i(j)$ to the traffic at k destined for j . Thus, for $(i, k) \in \mathcal{L}$, $i \neq j$

$$\begin{aligned} \frac{\partial U}{\partial \phi_{ik}(j)} &= t_i(j) \left[p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k) + \frac{\partial U}{\partial r_k(j)} \right] \\ &= t_i(j) \delta_{ik}(j). \end{aligned} \quad (8)$$

To summarize above discussions, we have the following theorem.

Theorem 1: Let a network have traffic input set \mathbf{r} and routing variables ϕ , and let each marginal utility $U_i'(p_i)$ be continuous in p_i , $i \in \mathcal{N}$. Then we have the following:

- the set in (7) $i \neq j$ has a unique solution for $\partial U / \partial r_i(j)$;
- both $\partial U / \partial r_i(j)$ and $\partial U / \partial \phi_{ik}(j)$ ($i \neq j, (i, k) \in \mathcal{L}$) are continuous in \mathbf{r} and ϕ .

Now we proceed to show the necessary and sufficient conditions for the optimal solution of the maximum lifetime routing problem. Recall that the maximum network lifetime routing problem is to route the traffic so that the network lifetime can be maximized for a given traffic input. Here, the solution space ψ consists of all the possible routing variable sets ϕ . The conditions for ϕ to be an optimal solution of the maximum lifetime routing problem are given in Theorem 2.

Theorem 2: Assume that U_i is concave and continuously differentiable for $p_i \forall i$. U is maximized if and only if for $\forall i, j \in \mathcal{N}$

$$\frac{\partial U}{\partial r_i(j)} \begin{cases} = \delta_{ik}(j), & \text{if } \phi_{ik} > 0 \\ \geq \delta_{ik}(j), & \text{if } \phi_{ik} = 0 \end{cases} \quad (9)$$

Theorem 2 states that the aggregate utility is maximized if at any node i , for a given commodity j , all links (i, k) that have any portion of flow $t_i(j)$ routed through ($\phi_{ik}(j) > 0$) must achieve the same marginal utility with respect to j , and that this maximum marginal utility must be greater than or equal to the marginal utilities of the links with no flow routed ($\phi_{ik}(j) = 0$).

²To simplify the notation $U_i'(p_i) = \partial U / \partial p_i$.

V. DISTRIBUTED ROUTING

A. Overview

By understanding the optimality conditions to maximum lifetime routing, the design philosophy of our routing scheme should now be clear. The algorithm works in an iterative fashion. In each iteration, for each node i and a given commodity j , i must incrementally decrease the fraction of traffic on link (i, k) (by decreasing $\phi_{ik}(j)$) whose marginal utility $\delta_{ik}(j)$ is large, and do the reverse for those links whose marginal utility is small, until the marginal utilities of all links carrying traffic are equal. When this condition is met for all nodes and all commodities, the entire system reaches the optimal point.

Therefore, for each node i , each iteration involves two steps, namely, 1) the calculation of marginal utility U'_{ik} for each outgoing link (i, k) and each of its downstream neighbors k 's marginal utility $\partial U / \partial r_k(j)$; and 2) the adjustment of routing variables $\phi_{ik}(j)$ based on the values of U'_{ik} and $\partial U / \partial r_k(j)$. We will elaborate them in details as follows.

Section V-B introduces how the calculation and update of marginal utilities U'_{ik} and $\partial U / \partial r_k(j)$ are executed. Section V-C discusses how to maintain loop-free routing. Section V-D formally presents the algorithm, whose optimal property is analyzed in Section V-E.

B. Calculation of Marginal Utilities

We first introduce how to calculate the link marginal utility $U'_{ik} = p_{ik}^t U'_i(p_i) + p_{ik}^r U'_k(p_k)$. Sending data over a wireless link (i, k) requires power consumption of both sending node i and receiving node k . Thus, the calculation of U'_{ik} depends on the cooperation of both nodes. Node i is responsible to calculate the term $p_{ik}^t U'_i(p_i)$. $U'_i(p_i)$ can be derived based on (6) if the energy reserve E_i and power consumption rate p_i are known. Both values can be directly measured by node i . p_{ik}^t can be calculated based on (3) if the constants α , β , and m , and the node distance d_{ik} , are known beforehand. Alternatively, node i can directly estimate p_{ik}^t by measuring the amount of data sent from i to k and the corresponding power consumption. Node k is responsible for calculate the term $p_{ik}^r U'_k(p_k)$. $U'_k(p_k)$ can be calculated in the same way as $U'_i(p_i)$. p_{ik}^r can be either calculated based on (2), or directly estimated by measuring the amount of data received at node k and the corresponding power consumption. After calculation, k can send the value of $p_{ik}^r U'_k(p_k)$ to node i , which in turn acquires U'_{ik} .

Now we see how each node i calculates its marginal utility $\partial U / \partial r_i(j)$ with respect to commodity j . In order to do so, based on (7), i needs to know $\delta_{ik}(j) = U'_{ik} + \partial U / \partial r_k(j)$, the marginal utility of all its outgoing links regarding commodity j . We have just discussed how to calculate U'_{ik} , and $\partial U / \partial r_k(j)$ is the marginal utility of i 's downstream neighbor k . Now it is clear that $\partial U / \partial r_i(j)$ should be calculated in a recursive way. Starting from node j , the recipient of commodity j , $\partial U / \partial r_j(j) = 0$ based on the definition. j then sends the values of $\partial U / \partial r_j(j)$ and $p_j^r U'_j(p_j)$ to its upstream neighbor, say k . Upon receiving the updates, node k can calculate U'_{ik} as described above, then acquire $\partial U / \partial r_k(j)$. Then, k repeats the same procedure to its upstream neighbor until node i is reached.

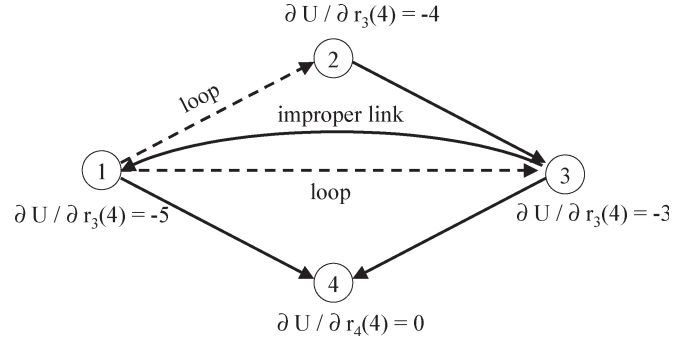


Fig. 2. Illustration of improper link.

C. Loop-Free Routing

From the above calculation, we can see that among all nodes carrying traffic of commodity j , their marginal utilities follow a partial ordering. The recipient node of commodity j has the highest marginal utility, which is 0. Its upstream neighbors have lower marginal utilities,³ whose own upstream neighbors have even lower marginal utilities. Therefore, the recursive procedure of node marginal utility calculation is free of deadlock if and only if such a partial ordering is maintained, i.e., the routing variable set ϕ is loop free.

In order to achieve loop-free routing, for each node i , with respect to commodity j , we introduce a set $B_{i,\phi}(j)$ of blocked nodes k for which $\phi_{ik}(j) = 0$ and the algorithm is not permitted to increase $\phi_{ik}(j)$ from 0. $k \in B_{i,\phi}(j)$ if one of the following conditions is met.

- 1) $(i, k) \notin \mathcal{L}$, i.e., k is not a neighbor of i .
- 2) $\phi_{ik} = 0$ and $\partial U / \partial r_i(j) \geq \partial U / \partial r_k(j)$, i.e., the marginal utility of k is already greater than or equal to the marginal utility of i .
- 3) $\phi_{ik} = 0$ and $\exists (l, m) \in \mathcal{L}$ such that (a) $l = k$ or l is downstream of k with respect to commodity j ; (b) $\phi_{lm}(j) > 0$ and $\partial U / \partial r_l(j) \geq \partial U / \partial r_m(j)$, i.e., (l, m) is an improper link.

An example illustrating an improper link is shown in Fig. 2. The solid line indicates that there is traffic on this link, and the dotted line indicates otherwise. Here, node 4 is the destination, and all the other nodes have input traffic destined to 4. The partial ordering of their marginal utilities are $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$, where the marginality of node 4 is the highest. However, the traffic from node 3 to node 1 flows against such partial ordering (node 3 has a higher marginal utility than node 1). Node 2, if unaware of the existence of such an improper link downstream, might make a loop by moving some of its outgoing traffic to node 3. To prevent this case from happening, node 3 only needs to raise a flag when updating its marginal utility to its upstream nodes 1 and 2. Upon receiving such a notification, nodes 1 and 2 can include node 3 into their blocking sets.

D. Algorithm

Now we are ready to formalize our algorithm. We use $\phi^{(k)}$ to represent the routing variable set at the iteration k . $\Delta\phi^{(k)}$ represents the changes made to $\phi^{(k)}$ during the iteration k .

³Given the definition of marginal utility in (7), its value is nonpositive.

Apparently, $\phi^{(k+1)} = \phi^{(k)} + \Delta\phi^{(k)}$. Also, for node i , we have the following.

- $\phi_i(j) = (\phi_{i1}(j), \dots, \phi_{im}(j))^T$ is the vector of its routing variable regarding commodity j .
- $\Delta\phi_i(j) = (\Delta\phi_{i1}(j), \dots, \Delta\phi_{im}(j))^T$ is the vector of changes to $\phi_i(j)$.
- $\delta_i(j) = (\delta_{i1}(j), \dots, \delta_{im}(j))^T$ is the vector of marginal utilities of all i 's neighbors.

At iteration k , node i operates according to the following steps.

- 1) Calculate the link marginal utility U'_{ik} for each of its going links (i, k) , get updates of marginal utility $\partial U / \partial r_k(j)$ from each of its downstream neighbors k , and then calculate $\delta_{ik}(j) = U'_{ik} + \partial U / \partial r_k(j)$.
- 2) Calculate its own marginal utility $\partial U / \partial r_i(j)$ according to (7) and send it to all its upstream neighbors.
- 3) Calculate $\Delta\phi_i^{(k)}(j)$ as

$$\Delta\phi_{il}^{(k)}(j) = \begin{cases} -\min\left\{\phi_{il}^{(k)}(j), \frac{\rho(\delta_{il}(j) - \delta_{\min}(j))}{t_i(j)}\right\}, & \delta_{il}(j) \neq \delta_{\min}(j) \\ \sum_{\delta_{im}(j) \neq \delta_{\min}(j)} \Delta\phi_{im}^{(k)}(j), & \delta_{il}(j) = \delta_{\min}(j) \end{cases}$$

where $\delta_{\min}(j) = \min_{m \in B_{i, \phi^{(k)}(j)}} \delta_{im}(j)$, and $\rho > 0$ is some positive stepsize.

- 4) Adjust routing variables

$$\phi_i^{(k+1)}(j) = \phi_i^{(k)}(j) + \Delta\phi_i^{(k)}(j), \quad \forall i \in \mathcal{N} - \{j\}.$$

E. Analysis

The following lemma shows some of the properties of our algorithm.

Lemma 2:

- 1) If $\phi^{(k)}(j)$ is loop free, then $\phi^{(k+1)}(j)$ is loop free.
- 2) If $\phi^{(k)}(j)$ is loop free and $\Delta\phi^{(k)}(j) = 0$ solves the problem defined in step (3) of the algorithm, then $\phi^{(k)}(j)$ is optimal.
- 3) If $\phi^{(k)}(j)$ is optimal, then $\phi^{(k+1)}(j)$ is also optimal.
- 4) If $\Delta\phi^{(k)}(j) \neq 0$ for some i for which $t_i(j) > 0$, then

$$U\left(\phi^{(k)}(j) + \Delta\phi^{(k)}(j)\right) > U\left(\phi^{(k)}(j)\right).$$

The following theorem shows the main convergence result.

Theorem 3: Let the initial routing $\phi^{(0)}$ be loop free and satisfy $U(\phi^{(0)}) \geq U_0$, where U_0 is some scalar, then

$$U\left(\phi^{(k+1)}(j)\right) \geq U\left(\phi^{(k)}(j)\right) \lim_{k \rightarrow \infty} U\left(\phi^{(k+1)}(j)\right) = \min_{\phi(j) \in \Phi(j)} U(\phi(j)).$$

Furthermore, every limit point of $\{\phi^{(k)}\}$ is an optimal solution to the problem defined in step (3) of the algorithm.

VI. SIMULATION STUDIES

A. Simulation Setup

We evaluate the performance of our routing algorithm via simulation in this section. Our simulation setting is as follows. We randomly create 100 nodes on a 100×100 m² square. The maximum transmission range of each node is 25 m. In our simulation, we set $\alpha = 50$ nJ/b, $\beta = 0.0013$ pJ/b/m⁴, and $m = 4$ for the power consumption model. The energy reserve on each node is 50 kJ.

We study two types of networking scenarios, namely, 1) sensor network and 2) *ad hoc* network. In the scenario of sensor network, a node is picked as the base station (data sink), while a subset of other nodes act as data sources sending traffic to the sink at 0.5 kb/s. The rest of the nodes act as relaying nodes. In the scenario of *ad hoc* network, we randomly create several pairs of unicast connections. Besides the senders and receivers of these unicast pairs, other nodes are responsible to relay traffic. The sending rate of each connection is 0.5 kb/s. We run each simulation over 20 different random topologies. For example, when evaluating the network lifetime of the sensor network scenario with 40 sensors, we create 20 different 100-node topologies and pick 40 nodes as data sources. We run algorithms on each of them and then show the average result.

We compare the performance of following algorithms. The MinEnergy algorithm tries to minimize the energy consumption for each data unit routed through the network. For each data source, the algorithm finds its shortest path to the destination in terms of energy cost. The route for each data source is fixed throughout the entire network lifetime. The MaxLife algorithm tries to maximize the network lifetime, which can be implemented in centralized or distributed fashions. The centralized algorithm derives the maximum lifetime by solving the linear programming problem. The distributed algorithm is the one presented in Section V. As mentioned in Section III, our distributed algorithm can only converge to near-optimal routing unless $\gamma \rightarrow \infty$. So we also evaluate the performance of our algorithm when γ takes different values.

B. Network Lifetime

Fig. 3(a) shows the lifetime of the same sensor network when the data are routed by different algorithms under the single-sink setting. We observe that the maximum lifetime of the network drops at a super-linear speed as the number of data sources increases, mainly as a result of increased traffic demand in a network with fixed overall energy reserve. It also shows that the optimal (centralized) MaxLife algorithm consistently maintains the network lifetime at least five times the same result returned by the MinEnergy algorithm. Also, the performance of our distributed algorithm is able to approximate the performance of the optimal algorithm by an average of 80% when $\gamma = 3$. When $\gamma = 4$, the average approximation ratio increases to 95%.

From Fig. 3(b), we have the same observations except that in the scenario of *ad hoc* network our algorithm outperforms the MinEnergy algorithm by only three times. This is mainly due to the different traffic patterns of two networking scenarios. In the sensor network scenario, the MinEnergy algorithm ends up with a shortest-path data aggregation tree, where the entire

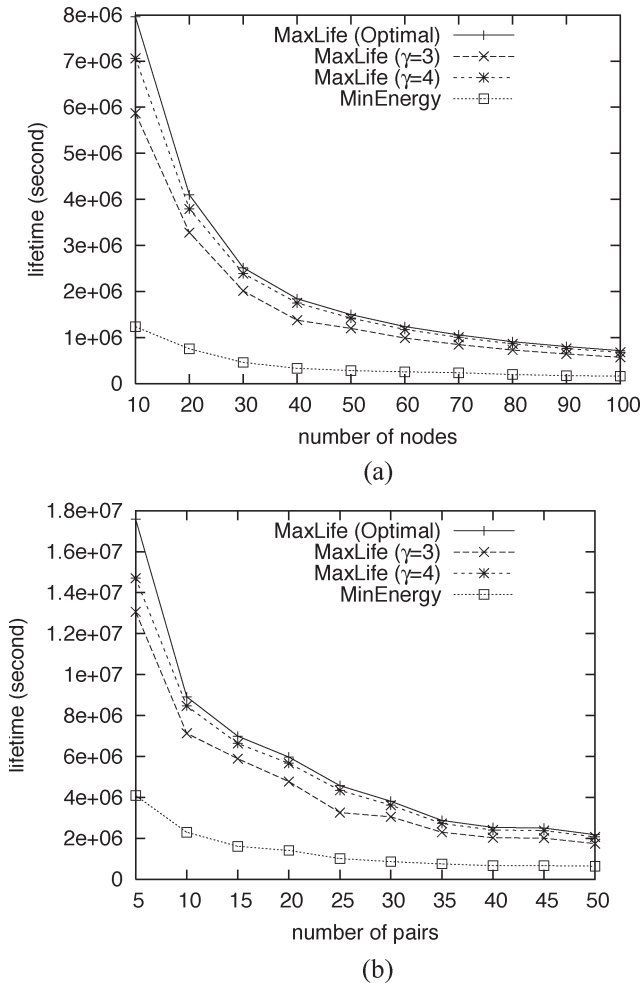


Fig. 3. Network lifetime. (a) Sensor network. (b) *Ad hoc* network.

traffic concentrated on a few nodes located close to the data sink. The energy reserve of these nodes can easily run out soon, which is the main reason for the inferior performance of this algorithm. The same traffic concentration problem exists in an *ad hoc* network scenario, but not as serious as the previous case. In contrast, in both networking scenarios, our algorithm is able to effectively diverge the traffic, hence the energy consumption, among all nodes, that significantly prolongs the network lifetime.

C. Energy Cost

On the other hand, our algorithm consumes more system energy than the MinEnergy algorithm for an average bit of data routed through the network. The reason is that, in order to maximally utilize the energy reserve of all nodes within the network, sometimes the data from a source have to go through some route whose energy consumption rate is not as efficient as the one returned by the MinEnergy algorithm. As shown in Fig. 4, such an inefficiency is bounded by a factor of 2 under various simulation settings.

D. Distribution of Energy Consumption

The distinction of the two algorithms' energy consumption pattern is further exhibited in Fig. 5, which plots the distribution

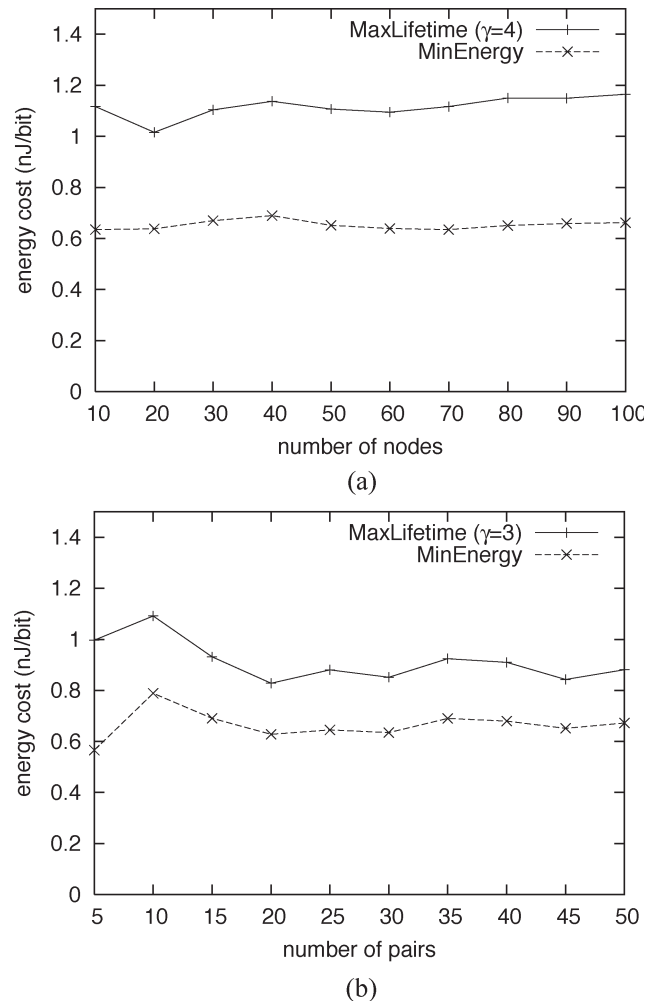


Fig. 4. Average energy cost. (a) Sensor network. (b) *Ad hoc* network.

of the energy consumption ratio of each node in the network throughout the entire lifetime. In both networking scenarios, 50 out of 100 nodes send traffic. For the MinEnergy algorithm, the distribution is highly asymmetric. Under both networking scenarios, only a few “hot spot” nodes completely utilize their energy reserves, while about 40% of the nodes do not consume any energy at all since they are not included as data relaying nodes.

On the other hand, in the result of our algorithm, most nodes get to contribute about 60% of its energy reserve since in our algorithm each node always tends to allocate more traffic via the node with the maximum marginal utility, i.e., the node with the least power consumption ratio. Thus, our algorithm is able to sustain a much longer system lifetime at the price of more energy consumption per bit than the MinEnergy algorithm.

VII. RELATED WORKS

Besides work on energy-efficient routing algorithms as we have discussed in Section I, there are other research efforts to address the constrained-energy problem in wireless networks.

Some works explore the performance limits of energy-constrained wireless networks. In particular, Hu and Li [28] study the energy-constrained fundamental limits with respect to

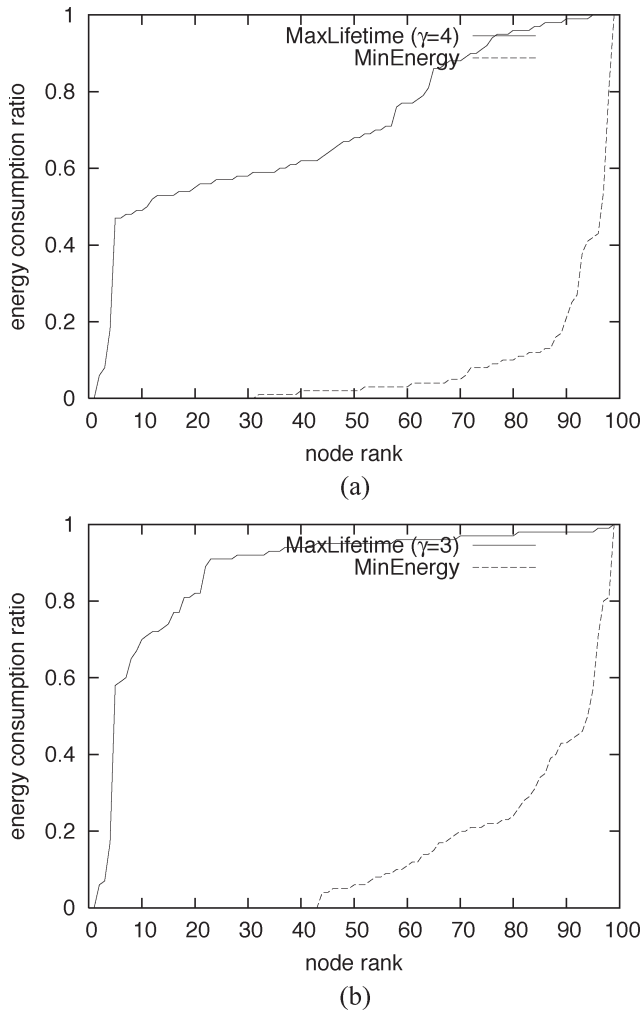


Fig. 5. Energy consumption distribution. (a) Sensor network (50 data sources). (b) Ad hoc network (25 unicast pairs).

network throughput and lifetime in wireless sensor networks. They give asymptotic analytical results on the relationship between network lifetime and the number of nodes with fixed node density. Zhang and Hou [24] derive the necessary and sufficient conditions of the node density in order to maintain k -coverage and the upper bound of network lifetime when only an α portion of a region is required to be covered, given a fixed node density in this finite region. Our work is different from these works in that we study how to achieve the upper bound of network lifetime via distributed routing instead of exploring what is this upper bound.

Hou *et al.* [29] study the problem of rate allocation with the requirement of network lifetime. The work in [30] also presents a rate allocation algorithm based on traffic splits in *ad hoc* networks. Our algorithm addresses a different problem from these works and may fit in different network operation environments. In particular, in our problem, the traffic demands from all sensors are fixed and known *a priori*. This problem well models the application scenarios, such as temperature, pressure, and noise level monitoring, where a fixed amount of information is generated at a fixed interval. Our goal is to maximize the network lifetime while satisfying the rate

demands instead of allocating rates to different wireless nodes such that certain fairness criteria are satisfied.

The presented distributed routing algorithm in this paper is similar to the works in [21] and [22] in that both algorithms explore the marginal utility (delay in their cases) to achieve the optimum in a distributed way. Yet this work studies a different network problem than these previous works. First, the goal of this work is to maximize the network lifetime, while the works in [21] and [22] are to minimize the aggregate delay of the network. Second, this work studies wireless nodes with energy constraint, while they study wireline links that incur delay when overloaded.

VIII. CONCLUSION

This paper studies the problem of distributed maximum network lifetime routing for a multihop wireless network. Inspired by the max-min fair resource allocation, this paper presents a novel utility-based nonlinear formulation of this problem. Based on this formulation, this paper further presents a distributed routing algorithm that achieves the goal of maximizing network lifetime. The presented algorithm has a solid nonlinear optimization theoretical background and is shown to be effective and efficient under various simulation environments.

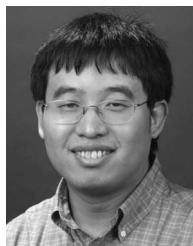
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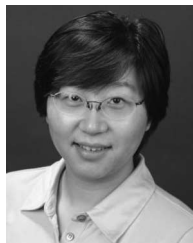
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